

## **Title: Gravel Roads and Sinusoidal Patterns**

### **Brief Overview:**

The student will construct and analyze sinusoidal (trigonometric) functions from given tables of gravel road erosions data, and use the graphing capabilities of the TI-83 calculator to compare best fit model to the data. Students will encounter real-world data that can be modeled closely with a sinusoidal graph even though the regression capabilities of a graphing calculator will not produce the most accurate model.

### **NCTM 2000 Principles for School Mathematics:**

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

### **Links to NCTM 2000 Standards:**

- **Content Standards**

- Number and Operations**

- Students will compute fluently and make reasonable estimates.

- Algebra**

- Students will understand patterns, relations, and functions and use mathematical models to represent and understand quantitative relationships.

- Data Analysis and Probability**

- Students will use scatter plots to display data, identify trends, and find functions that model the data.

- **Process Standards**

- Problem Solving**

- Students will build mathematical knowledge of sinusoidal curves through problems and apply and reflect upon the process of applying strategies to mathematical problem solving.

- Reasoning and Proof**

- Students will be able to recognize patterns among and construct functions that fit data. In the process making and investigating mathematical conjectures, they will also be able to explain and justify the components of the functions they construct.

- Communication**

- Students will organize their mathematical thinking and communicate their thinking coherently and clearly using functions and graphical representations of data and verbal or written explanations. Students will also evaluate the mathematical thinking and strategies of others. In addition, students will use the language of mathematics to express mathematical ideas precisely.

- Connections**

- Students will recognize and use connections among mathematical ideas and recognize and apply mathematics in contexts outside of mathematics.

- Representation**

- Students will organize, record and communicate mathematical ideas and select and apply mathematical representations to model and interpret physical, social and mathematical phenomena.

**Links to Maryland High School Mathematics Core Learning Units:**

- Functions and Algebra**

- **1.1.1**

- Students will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2**

- Students will represent patterns and functional relationships in a table, as a graph and/or by mathematical expression.

- **1.1.4**

- Students will describe the graph of a non-linear functions in terms of the basic concepts of maxima and minima, roots, limits, rate of change and continuity.

- **1.2.4**

- Students will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

## **Data Analysis and Probability**

- **3.1.1**

Students will design and/or construct an investigation that uses statistical methods to analyze data and communicate results.

- **3.2.1**

Students will make informed decisions and predictions based upon the results of simulations and data from research.

### **Grade/Level:**

Grades 11-12, Trigonometry, Pre-Calculus

### **Duration/Length:**

Two 45 minutes periods (one for each activity) plus time for assessment. Activity 1 provides a review the concepts and re-introduces/reviews data entry and creation of a scatter plot using a TI-83 or TI-83 Plus (collectively referred to hereinafter as a “TI-83”). The Addendum to Activity 1 may be used for additional practice or homework. Activity 2 focuses on the same topics using measurements from gravel roads. The Addendum to Activity 2 can be used as homework or further in-class work/review. The Assessment Section can be used during an additional period or part thereof or be assigned as homework.

### **Prerequisite Knowledge:**

Students should have working knowledge of the following skills:

- Inputting data into a TI-83 and creating a scatter plot
- Recognizing and identifying graphs of sinusoidal functions (sine and cosine)
- Identifying maximum and minimum function values
- Identifying the amplitude, period and vertical shift of basic sine and cosine functions
- Translating sine and cosine functions including an understanding of the impact coefficients  $a$ ,  $b$  and  $d$  have on the functions:  $f(x) = a \cos(bx) + d$ ,  $f(x) = a \sin(bx) + d$

### **Student Outcomes:**

Students will:

- interpret given data.
- calculate amplitude, period and vertical shift of a sine or cosine function given the graph of the function.
- write functions in the form  $f(x) = a \cos(bx) + d$  or  $f(x) = a \sin(bx) + c$  for the graph of a sine or cosine function by determining the values of coefficients  $a$ ,  $b$ , and  $c$ .
- fit a sinusoidal function to data without using the regression capabilities of the TI-83.

**Materials/Resources/Printed Materials:**

- Student Worksheets: Activity 1, Activity 2, Addenda for Activities 1 and 2 (*optional*) and the Assessment
- TI-83 graphing calculator

**Development/Procedures:**

The teacher should review graphs of sine and cosine curves including the impact of varying coefficients  $a$ ,  $b$  or  $d$  in the graphs  $f(x) = a \cos(bx) + d$  and  $f(x) = a \sin(bx) + d$ . To prepare for the application to data gathered from taking measurements along gravel roads, the teacher should lead the class in a discussion of unpaved gravel or crushed rock/stone covered roads or driveways. This should be followed by an explanation of the "washboard" phenomenon to allow students to have a visual image of the issue. Possible discussion topics could include identifying similar roads in the area, discussing why municipalities or individuals use gravel instead of paving (aesthetics, cost, maintenance, etc.), and asking for suggestions of other washboard images: ripples on a pond, or left in sand by waves or winds in desert areas. Teachers may want to have a washboard as a visual aid to help their students understand the application.

**Assessment:**

Students should use the skills covered in their learning units to analyze data that can be modeled by a sinusoidal function to find key coordinates for such a function. The assessment may be given during a class period or given as a homework assignment if students have access to a TI-83.

**Extension/Follow Up:**

- Students should be encouraged to think of another real-world situation that can be model by a sinusoidal curve, collect data (either physically or via research) and perform a similar analysis.
- Teachers may want to discuss how both a sine and cosine curve can be used to model the data by including the appropriate horizontal shift coefficient in the function.

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## INTERPRETING SINUSOIDAL FUNCTIONS FROM GRAPHS

### ACTIVITY 1: THE SINUSOIDAL FUNCTION

#### I. REVIEW OF THE SINUSOIDAL FUNCTION

The sinusoidal function can be used to describe many naturally occurring phenomena such as earthquakes, sound waves, electrical current, and the movement of waves in a lake or ocean. Any data in a repeating wave-like pattern can be simulated using a function involving sine or cosine.

The basic sinusoidal function is either:

$$f(x) = a \sin(b(x + c)) + d \text{ or } f(x) = a \cos(b(x + c)) + d$$

The *coefficients*  $a$ ,  $b$ ,  $c$ , and  $d$  variables are defined as follows:

$$a = \text{amplitude} \left( \left| \frac{\text{maximum} - \text{minimum}}{2} \right| \right)$$

$b$  = the coefficient used to determine the length of the period ( $\frac{2\pi}{b}$  = period length)

$c$  = horizontal or phase shift

$d$  = vertical shift  $\left( \frac{\text{maximum} + \text{minimum}}{2} \right)$ . As a result of the vertical shift  
 $y = d$  replaces  $y = 0$  as the *horizontal axis*

Example: Given the function  $y = 3 \sin(2\pi x) - 1$  determine the:

Amplitude \_\_\_\_\_ Period length \_\_\_\_\_

Horizontal shift \_\_\_\_\_ Vertical shift \_\_\_\_\_

Maximum function value \_\_\_\_\_ Minimum function value \_\_\_\_\_

## SOLUTIONS:

Amplitude = 3    Period length = 1    Horizontal shift = none    Vertical shift = -1

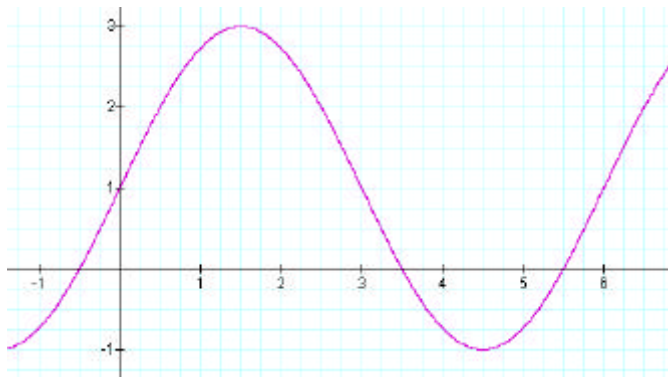
Maximum = 2 → sin x has a maximum value of 1 multiplied by the amplitude, 3, then shifted down 1 because  $d = -1$ .

Minimum = -4 → sin x has a minimum value of -1 multiplied by the amplitude, 3, then shifted down 1 because  $d = -1$ .

## II. DETERMINING COEFFICIENT VALUES FROM A GRAPH OF THE FUNCTION

In this exercise a graph is presented to demonstrate the procedure used to determine a function that could be used to represent the data shown. Since both *sine* and *cosine* are periodic functions, the starting point for the function is usually selected as the *easiest* location to determine the necessary values. **Remember**, the cosine model works best when the model begins on a maximum or minimum point whereas the sine model works best when the model begins on the horizontal axis.

**Graph 1.**



### A. DETERMINING THE COEFFICIENT VALUES NEEDED TO CREATE THE FUNCTION.

*Show all your work in the space given for your calculations.*

- 1) Determine the maximum and minimum values of the function.

Maximum = \_\_\_\_\_ Minimum = \_\_\_\_\_

- 2) Use the y values from the coordinates of consecutive maximum and minimum points to determine the amplitude ( $a$ ).

Calculations:

$a =$  \_\_\_\_\_

- 3) Use the maximum and minimum to determine the horizontal axis (vertical shift),  $d$ .  
Calculations:

$d =$  \_\_\_\_\_ Horizontal axis  $y =$  \_\_\_\_\_

- 4) Determine the period length. The period length can be determined: (i) using the  $x$  values to calculate the distance between two maximum points; (ii) twice the distance between a maximum and a minimum; or (iii) the sum of the distances between three points where the graph crosses the horizontal axis. The period length is used to determine the value of  $b$ .

Period length = \_\_\_\_\_      Period =  $\frac{2p}{b}$

Calculate the value of  $b$ .

Calculations:

$$b = \underline{\hspace{2cm}}$$

- 5) This function can be started anywhere but by starting on the y-axis the need for a horizontal shift is avoided.

If the y-axis was chosen as the initial point of the graph, would the graph be best represented using *sine* or *cosine*. \_\_\_\_\_

- 6) Construct your equation using the calculated values for  $a$ ,  $b$ , and  $d$ .

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- 7) Use your calculator (in the correct mode) to check your graph with the one given above.  
Set your window as shown.

WINDOW
Xmin=0
Xmax=7
Xscl=1
Ymin=-4
Ymax=4
Yscl=1
Xres=1

- 8) Does your graph match the given graph? \_\_\_\_\_. If not, determine from your graph which coefficients are incorrect.

**B. For more practice with writing equations to match graphs, see the addendum.**

### III. DETERMINING COEFFICIENT VALUES FROM A TABLE OF DATA

#### A. Data




$x$	$y$
0	4
.5	-2
1	-6.29
1.5	-8
2	-6.39
2.5	-2
3	4
3.5	10
4	14.54
4.5	16
5	14.39

#### 1. ENTERING DATA INTO A TI-83

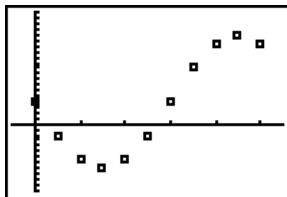
- Clear lists  $L_1$ ,  $L_2$ ,  
**STAT**, **ClrList**,  $L_1$ ,  $L_2$ , **ENTER**.
- Enter data into list.
- Press **STAT**.
- Press **1:Edit**.
- Place  $x$  values in  $L_1$ .
- Place  $y$  values in  $L_2$ .

L1	L2	L3	3
0	4		
.5	-2		
1	-6.29		
1.5	-8		
2	-6.39		
2.5	-2		
3	4		
L3(1)=			

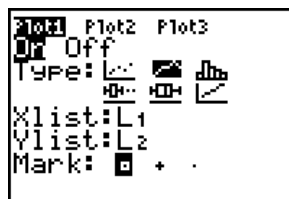
- Turn on **STATPLOT** [**2<sup>nd</sup> Y=**].
- Select **PLOT 1**.
- Set to graph scatter plot.

<b>2001</b>	P1ot2	P1ot3
<b>On</b>	Off	
Type:		
Xlist:	L1	
Ylist:	L2	
Mark:		+

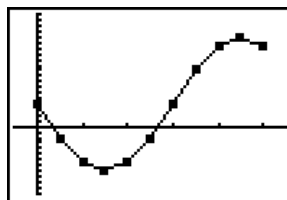
j) Press **ZOOM 9**.



k) Reset **STATPLOT**  
to graph a connected plot.



l) Press **GRAPH**.



## 2. CALCULATING CONSTANT VALUES FROM DATA

*Show all your work in the space given for your calculations.*

a) Determine values of the maximum and minimum.

Maximum = \_\_\_\_\_ Minimum = \_\_\_\_\_

b) Calculate amplitude from maximum and minimum values.

Calculations:

$a =$  \_\_\_\_\_

- c) Calculate the horizontal axis from the maximum and minimum,  $d$ .  
Calculations:

$d =$  \_\_\_\_\_ Horizontal axis  $y =$  \_\_\_\_\_

- d) Determine the period length from the maximum and minimum.  
Calculations:

Period length = \_\_\_\_\_ Period =  $\frac{2p}{b}$   $b =$  \_\_\_\_\_

- e) If the y-axis were chosen as the initial point of the graph, would the graph be best represented using *sine* or *cosine*? \_\_\_\_\_.
- f) Construct the function using the calculated values of  $a$ ,  $b$ , and  $d$ .

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- Enter the equation into **Y1**. Move your cursor to the left of Y1 and press enter to select the **bold** line. Make sure the equal sign is highlighted.
- Press **GRAPH**.
- Compare the graph of the equation determined with the graph of the data. Discuss the similarity between the two graphs. Are they similar, identical, or very dissimilar? Explain.

- j) If the graphs don't match, recheck the coefficients you calculated and try again.
- k) Once you have a close match, compare your equations with other students. Be able to justify your solution using your calculations.

**B. For additional in-class practice or additional homework, more data sets are included in the addendum.**

## ACTIVITY 1 ANSWER SHEET

### DETERMINING COEFFICIENT VALUES FROM A GRAPH OF THE FUNCTION

1. Maximum = 3   Minimum = -1

2. Amplitude = 2  
 $a = 2$

3. Vertical shift = 1  
 $d = 1$

4. Period length = 6  
 $b = \frac{P}{3}$

5. The function is best represented with a sine function.

6.  $y = 2 \sin\left(\frac{P}{3}x\right) + 1$

### DETERMINING COEFFICIENT VALUES FROM DATA

### CALCULATING CONSTANT VALUES FROM DATA, QUESTION 2

a. Maximum = 16   Minimum = -8

b. Amplitude = 12  
 $a = -12$  (is negative because the graph starts by decreasing)

c. Horizontal axis is  $y = 4$   
 $d = 4$

d. Period length = 6  
 $b = \frac{P}{3}$

e. Equation  
 $y = -12 \sin\left(\frac{P}{3}x\right) + 4$

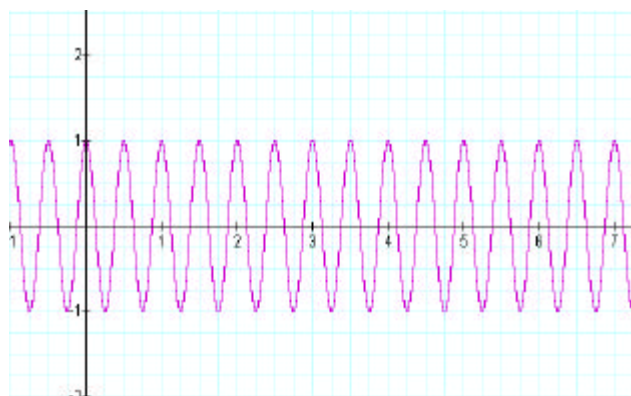
i. The two graphs are identical because the data was taken from a table determined by the function.

## INTERPRETING SINUSOIDAL FUNCTIONS FROM GRAPHS

### ADDENDUM - ADDITIONAL PRACTICE FOR ACTIVITY 1

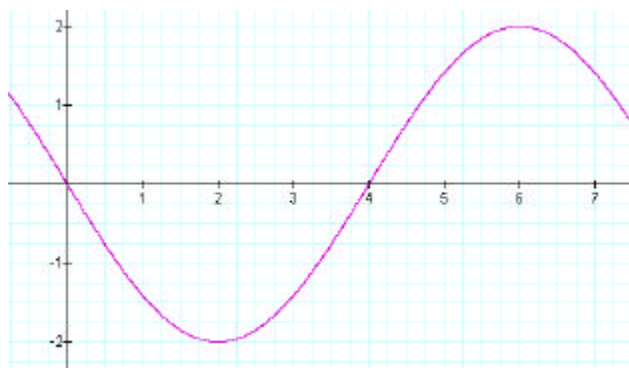
Determine the function pictured in each graph.

1.



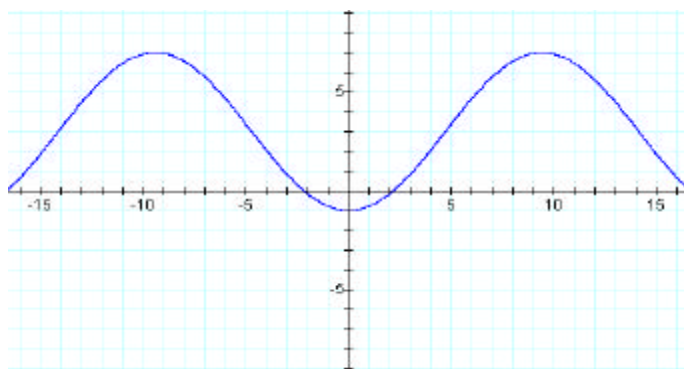
Equation \_\_\_\_\_

2.



Equation \_\_\_\_\_

3.



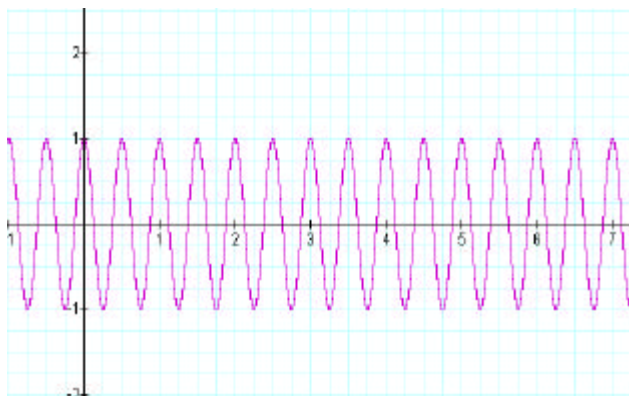
Equation \_\_\_\_\_

## INTERPRETING SINUSOIDAL FUNCTIONS FROM GRAPHS

### **ADDENDUM - ANSWERS FOR ADDITIONAL PRACTICE FOR ACTIVITY 1** **ADDITIONAL PRACTICE FOR ACTIVITY 1    ANSWERS**

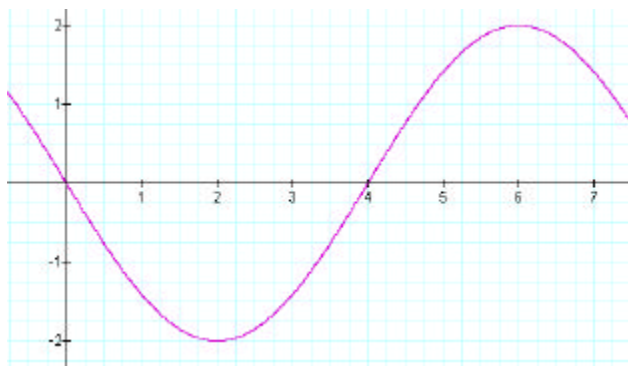
Determine the function pictured in each graph.

1.



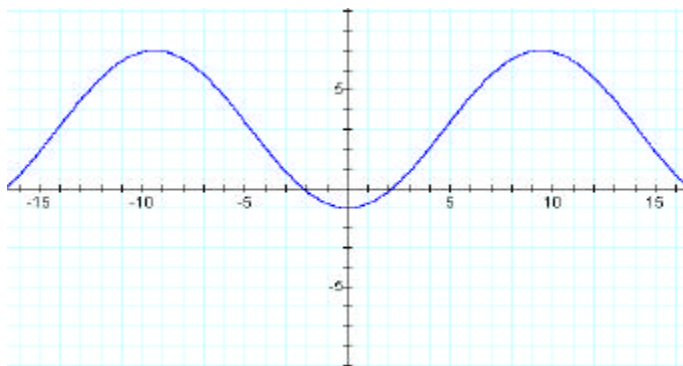
Equation  $y = \cos(4\pi x)$

2.



Equation  $y = -2 \sin\left(\frac{p}{4} x\right)$

3.



Equation  $y = 4 \cos\left(\frac{p}{9} x\right) + 3$

## **INTERPRETING SINUSOIDAL FUNCTIONS FROM DATA**

### **ADDENDUM - ADDITIONAL PRACTICE FOR ACTIVITY 1**

Use the data tables and the scatter plot to determine the function represented by the data.

1.

X	Y
0	5
4	3
8	-1
12	-3
16	-1
20	3
24	5
28	3

Equation \_\_\_\_\_

2.

X	Y
0	3
.1	1.61
.2	-1.75
.25	-2
.35	-1.05
.45	1.45
.5	3
.6	5.94
.75	8
.9	5.94

Equation \_\_\_\_\_

3

X	Y
0	10
.2	-2
.4	10
.6	-2
.8	10



**INTERPRETING SINUSOIDAL FUNCTIONS FROM DATA**  
**ANSWERS**

Use the data tables and the scatter plot to determine the function represented by the data.

1.

X	Y
0	5
4	3
8	-1
12	-3
16	-1
20	3
24	5
28	3

Equation  $y = 4\cos\left(\frac{p}{12}x\right) + 1$

2.

X	Y
0	3
.1	1.61
.2	-1.75
.25	-2
.35	-1.05
.45	1.45
.5	3
.6	5.94
.75	8
.9	5.94

Equation  $y = -5\sin(2px) + 3$

3

X	Y
0	10
.2	-2
.4	10
.6	-2
.8	10

Equation  $y = 6\cos(5px) + 4$

## INTERPRETING SINUSOIDAL FUNCTIONS FROM DATA

### ACTIVITY 2

#### THE WASHBOARD EFFECT ON GRAVEL ROADS

Gravel roads often show a pattern of erosion called “washboarding”. This is a wearing of the road surface creating a series of bumps that resemble the surface of an old-fashioned washboard. The data below was collected from Ingleside Road in Cheboygan County, Michigan. Use the data to create a sinusoidal formula that represents the washboard pattern of that road.

Horizontal distance was measured from a starting point atop one of the peaks. The vertical distances were measured as the distance above the normal road surface (the top of the peak) or below the normal road surface (the bottom of the valley).

Horizontal distance (cm)	Vertical distance (cm)
0	2
17	-1
40	2
60	-1.8
78	2.4
97	-2
115.5	2.7
129	-1
148	1.8
165	-1.2
181	1.7
197	-2
216	2
231	-1
253	2

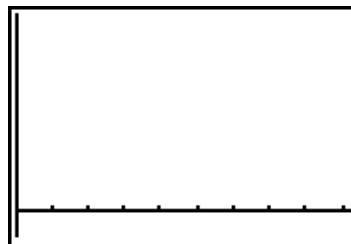
#### I. GRAPHING THE DATA

1. Clear lists  $L_1$ ,  $L_2$  STAT, **ClrList**,  $L_1$ ,  $L_2$ , **ENTER**.
2. Press **STAT**.
3. Enter Horizontal Data in  $L_1$ .
4. Enter Vertical Data in  $L_2$ .

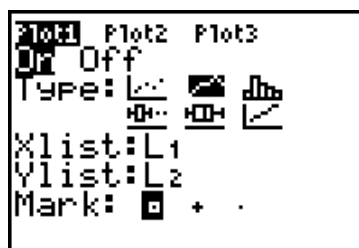
L1	L2	L3	1
0	2	-----	
17	-1		
40	2		
60	-1.8		
78	2.4		
97	-2		
115.5	2.7		
L1(1)=0			

5. Turn on **STAT PLOT**.
6. Press **ZOOM 9**.

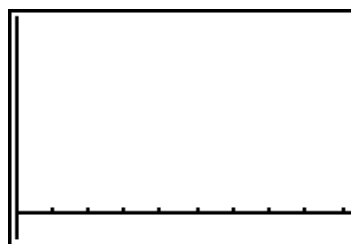
7. Sketch the **STAT PLOT** here.



8. Change **STAT PLOT** to connect the data points.



9. Draw the graph seen here.



## II. DETERMINING THE EQUATION

*Show all your work in the space given for your calculations.*

1. Is the data best represented using a sine or cosine function? (Use the starting point to make your determination.) \_\_\_\_\_
2. Determine the *average* maximum height. \_\_\_\_\_
3. Determine the *average* minimum height. \_\_\_\_\_]
4. Determine the *average* distance between maximums. \_\_\_\_\_
5. Use the average maximum and minimum to determine the amplitude ( $a$ ).  
Calculations:

$$a = \underline{\hspace{2cm}}$$

6. Use the average distance between maximums to determine the period length.  
Use the period length to determine  $b$ .  
Calculations:

$$\text{Period length} = \underline{\hspace{2cm}} \quad \text{Period} = \frac{2p}{b}$$

$$b = \underline{\hspace{2cm}}$$

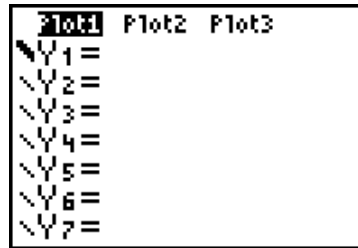
7. Use the average maximum and minimum to determine the vertical shift ( $d$ )  
Calculations:

$$d = \underline{\hspace{2cm}} \quad \text{Horizontal axis } y = \underline{\hspace{2cm}}$$

8. Use the calculated of  $a$ ,  $b$ , and  $d$  to construct the equation that fits the data.

\_\_\_\_\_

9. Enter the equation into **Y1=**.
10. Change line to bold.
11. Make sure the equal sign is highlighted.
12. Press **GRAPH**.
13. Compare the two graphs.



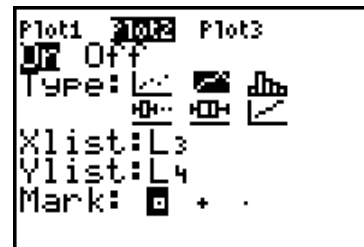
16. How similar is the graph of your equation to the actual data? (If they are not close, then recheck your calculations and try again.)
17. Compare your answer with your classmates.
18. Notice that the last data point was (253, 2) *measured in cm*. Assuming the model continues to hold true, how many "bumps" will you encounter over the next 1.1 m of this gravel road?

### III. A SECOND ROAD

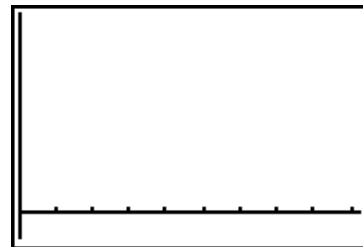
LOCATION: McArthur Rd

Distance (cm)	Height (cm)
0	2.3
20.4	-1.3
39.4	2.4
63.4	-1.1
86.2	2.4
107.2	-1
129	2.2
151.2	-1.4

1. Clear **L<sub>3</sub>** **L<sub>4</sub>**,  
**STAT**, **ClrList**, **L<sub>3</sub>**, **L<sub>4</sub>**, **ENTER**.
2. Enter the data for McArthur Rd. into **L<sub>3</sub>** and **L<sub>4</sub>**.
3. Turn off **PLOT 1** and turn on **PLOT2**.
4. Reset **XLIST** to **L<sub>3</sub>** in Plot 2.
5. Reset **YLIST** to **L<sub>4</sub>** in Plot 2.
6. Press **ZOOM 9**.
7. Leave on **Y1**, the model for Engleside road from section I above.



8. Sketch the two graphs.



9. Compare the new plot to the model for Engleside Road.
10. If there was any difference, what might account for it?

#### IV. ADDITIONAL PRACTICE FOR COMPLETION AT HOME

LOCATION: Brill Rd

Distance (cm)	Height (cm)
0	2.6
23.2	-1.1
42.9	2.4
65.5	-1.4
87.1	2.5
111.6	-1.7
126.6	2.2

Repeat the same procedure for the third road.

1. Enter the data for Brill Rd. into **L<sub>5</sub>** and **L<sub>6</sub>**.
2. Plot the data.
3. Compare the plot to the Model in **Y1**.

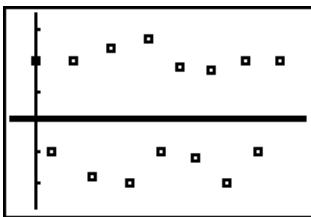
Compare the new plot and the model in **Y1**. If you found any difference, what might account for the difference?

## THE WASHBOARD EFFECT ON GRAVEL ROADS ANSWERS

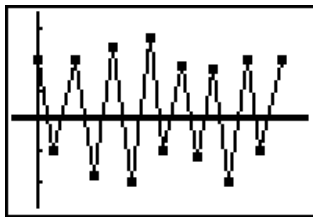
1. Data in calculator

L1	L2	L3	1
0	2	-----	
17	-1		
40	2		
60	-1.8		
78	2.4		
97	-2		
115.5	2.7		
L1()=0			

2. Scatter plot



3. Scatter plot with connected lines





4. Statistics

Average Maximum = 2.075 cm

Average Minimum = -1.429 cm

Average Distance Between Maximums = 36.143 cm

Amplitude  $a = 1.752$

Period length = 36.143

$$\frac{2p}{b} = 36.143$$

$$b = \frac{2p}{36.143}$$

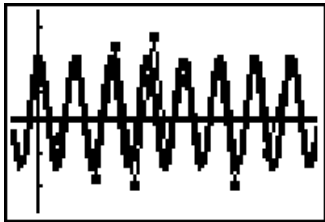
$$b = .174$$

Vertical Shift  $d = .323$

Equation from data

$$h = 1.75\cos(.174x) + .323$$

Graph from equation matched to data



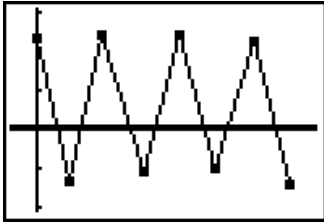
The number of bumps encountered in the next 1.1 m. after the last data point take at 253 cm.

$$\frac{2p}{b} = \frac{2p}{.174} = 36.110 \qquad 110 \div 36.11 \approx 3$$

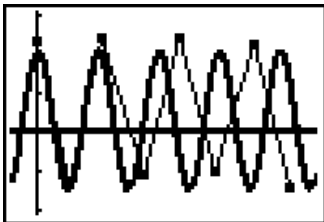
You will encounter 3 bumps.

## McArthur Road

### Scatter plot with points connected



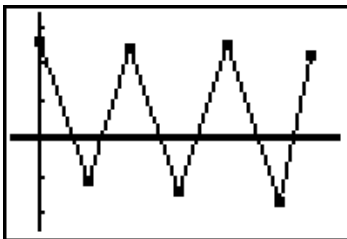
### Equation on scatter plot



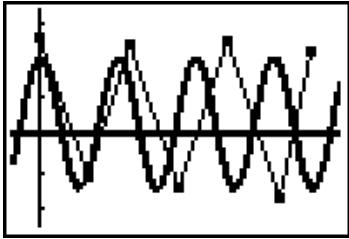
The two graphs are similar but not exact. Differences may be due to different sample sizes, slightly different gravel, or differences in the amount in traffic.

## Brill Road

### Scatter plot with points connected



### Equation on scatter plot



The scatter plot does not match exactly but is close. Maybe sample sizes should be increased or the data from the roads should be averaged together.

## **INTERPRETING SINUSOIDAL FUNCTIONS FROM GRAPHS AND DATA**

### **TEACHER'S GUIDE**

#### **Introduction**

This learning unit is designed to strengthen the student's ability to find the equation of a sinusoidal curve given either a graph of the curve or a set of data, without using the regression capabilities of a graphing calculator. In particular a student will learn how to find amplitude, period, and vertical shift and will learn how these values relate to the coefficients  $a$ ,  $b$ , and  $d$  in the functions:  $f(x) = a \sin(b(x + c)) + d$  and

$f(x) = a \sin(bx) + c$  For the purpose of finding amplitude we used the equation:

$\left( \left| \frac{\text{maximum} - \text{minimum}}{2} \right| \right)$ . We have assumed students know that the sign of  $a$  depends

on whether the sine or cosine function is increasing or decreasing. The activities provide a reminder/exploration should student have forgotten to give  $a$  the appropriate sign.

#### **Objectives Covered**

- Recognize data that can be modeled by a sinusoidal curve.
- Find the equation for a sinusoidal curve without using regression capabilities of a calculator.
- Use a sinusoidal model to make predictions.

#### **Tools/Materials Needed for Assessment**

- Assessment Sheet and Student Response Sheet
- TI-83 Calculator

#### **Administering the Assessment**

The assessment may be given during a class period or as a take home assignment provided students have access to a TI-83.

## INTERPRETING SINUSOIDAL FUNCTIONS FROM DATA

### ASSESSMENT

- Which function has maximum points at (6,1) and (12,1)?
  - $y = 6 \sin(6\pi x)$
  - $y = 5 \sin(\frac{p}{3}x) + 1$
  - $y = \cos(6\pi x) + 1$
  - $y = 5 \cos(\frac{p}{3}x) + 1$
- A sinusoidal function  $p(x)$  has a minimum value at (0, -4.5). The consecutive maximum value occurs at (3.5, .5). Find the amplitude of this function.
  - 2
  - 2.5
  - 2
  - 2.5
- $B(x) = 1.93 \cos(.149x) + .513$  is a function that models the washboard effect found on Brill Road in Cheboygan County, Michigan. Assume the model holds true for the first ten meters of this gravel road. If you begin a bike ride at the top of the first peak (bump) when distance  $x = 0$  cm, how many bumps will you ride over as you bike the first 5 meters of Brill Road. Grid in your answer on the answer sheet.

## **INTERPRETING SINUSOIDAL FUNCTIONS FROM DATA**

### **ASSESSMENT - continued**

4. **(BCR)** The sinusoidal coaster at Function Land amusement park was built according to the equation  $h = 16 \sin\left(\frac{P}{55}d\right) + 22$  where  $h$  is the height in feet of the coaster track and  $d$  represents the distance from the beginning.
- (a) If the entire ride covered 660 feet, how many times would the track reach its maximum height?
  - (b) How far into the ride would the first rapid descent occur?
5. **(ECR)** The Bay of Fundy in Nova Scotia has the highest tides in the world. The change in tide is approximately a sinusoidal function. High tides occur every 12 hours and 25 minutes. During the month of May, the average high tide was 38.7 ft. The average low tide was 5.8 ft.
- (a) Starting at time of high tide, determine an equation that models the tidal flow in the Bay of Fundy.
  - (b) In the month of May, what would the greatest number of times that the tide was at a high point?
  - (c) If a high tide occurred at 11:45 P.M., when would the next low tide and high tide occur?
  - (d) If a high tide occurred at 11:45 P.M., what would the height of the tide be at 9:00 A.M. the next morning.

## **ASSESSMENT - STUDENT RESPONSE SHEET**

**Name** \_\_\_\_\_

Date\_\_\_\_\_

1. Circle the correct answer.

**A.  $y = 6 \sin( 6p x )$**

**B.**  $y = 5 \sin(\frac{p}{3}x) + 1$

**C.**  $y = \cos(6\mathbf{p}\mathbf{x}) + 1$

**D.**  $y = 5 \cos(\frac{p}{3}x) + 1$

2. Circle the correct answer.

### A. 2

### B. 2.5

**C. -2**

### D. -2.5

3. Grid in your answer

[illegible]

**ASSESSMENT - STUDENT RESPONSE SHEET - Page 2**

**Name** \_\_\_\_\_

**Date** \_\_\_\_\_

4. Show your work here.

5. Show your work here.



## INTERPRETING SINUSOIDAL FUNCTIONS FROM DATA

### ASSESSMENT - ANSWER SHEET

1. Circle the correct answer.

A.  $y = 6 \sin(6\pi x)$

B.  $y = 5 \sin\left(\frac{p}{3}x\right) + 1$

C.  $y = \cos(6\pi x) + 1$

D.  $y = 5 \cos\left(\frac{p}{3}x\right) + 1$

2. Circle the correct answer.

A. 2

B. 2.5

C. -2

D. -2.5

3. 11

4. **BCR**

- There would be 6 peaks in the track.
- The first drop would occur 27.5 ft into ride.

5. **ECR**

Let  $h$  = height of the tide and  $t$  = time since start (hrs)

- **Equation**  $h = 16.45 \cos(.506x) + 22.25$
- Since high tides occur on average every 12 hr. 25 min. (12.417 hr.), 59-60 high tides would be expected during the month of May.
- The next low tide would occur at approximately 5:58 A.M. The next high tide would happen at approximately 12:10 P.M.
- The height of the tide at 9:00 A.M. would be approximately 22.03 ft.

## **INTERPRETING SINUSOIDAL FUNCTIONS FROM DATA**

### **ASSESSMENT - Scoring Guide**

For questions 1, 2, and 3, the teacher should assign his or her typical point values for multiple choice and "grid-in" problems.

Question # 4

**BCR**

<b>3 pts.</b>	<b>2pts.</b>	<b>1pt.</b>
Answers to both questions are correct and show a strong understanding of the topic.	Answers to the first part (number of peaks) is correct but the distance found for the second part is incorrect. Student shows good understanding but has made computation errors.	Answers are incorrect but student's process shows some understanding of the topics.

Question # 5

**ECR**

<b>4 pts.</b>	<b>3 pts.</b>	<b>2pts.</b>	<b>1pt.</b>
All answers are correct. The work shows a complete understanding of the material.	The equation is correct. At least three or the four remaining questions are correct, but the student's work is difficult to follow and does not show complete understanding of the material.	The equation is incorrect but the student has correctly determined two of the three constants. The student uses the equation to correctly answer the remaining questions.	Only one constant in the equation is correct. The remaining answers are incorrect according to the equation determined by the student.